The polar equation of an ellipse referred to the center as pole is

$$r^2=\frac{a^2b^2}{a^2\sin^2\theta+b^2\cos^2\theta}.$$

Since section $EAC=CAF$, and $DAF=DA'G$; we have for $ECFA$ integral

$$a^2b^2\int_{\frac{1}{2}\omega}^{\omega}\frac{d\theta}{a^2\sin^2\theta+b^2\cos^2\theta}=ab\left[\tan^{-1}\left(\frac{a}{b}\tan\omega\right)-\tan^{-1}\left(\frac{a}{b}\tan\tfrac{1}{2}\omega\right)\right]$$

$$=ab\tan^{-1}\left(\frac{ab\sin\omega}{a^2\sin^2\omega+b^2\cos^2\omega+b^2\cos\omega}\right).$$

To find the area of $AFDG$ we have only to put $\pi-\omega$ in place of ω, and thus we get

$$ab\tan^{-1}\left(\frac{ab\sin\omega}{a^2\sin^2\omega+b^2\cos^2\omega-b^2\cos\omega}\right).$$

$\therefore$ Area of $NCDM$

$$=2ab\left[\tan^{-1}\left(\frac{ab\sin\omega}{a^2\sin^2\omega+b^2\cos^2\omega+b^2\cos\omega}\right)+\tan^{-1}\left(\frac{ab\sin\omega}{a^2\sin^2\omega+b^2\cos^2\omega-b^2\cos\omega}\right)\right]$$

$$=2ab\tan^{-1}\left[\frac{2ab}{(a^2-b^2)\sin\omega}\right].$$

Also solved by *G. B. M. ZERR.*

115. Proposed by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College, Defiance, Ohio.

The axes of two right elliptic cylinders intersect at right angles in such a manner that the *major axes of the sections* are perpendicular. Supposing the axes to be $(A, B)>(a, b)$, what is the common volume ?

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let $x^2/B^2+z^2/A^2=1$, $x^2/a^2+y^2/b^2=1$, be the equations to the cylinders.

$$\therefore V=\frac{8Ab}{aB}\int_0^B\sqrt{[(a^2-x^2)(B^2-x^2)]}dx,\ B<a\ldots\ (1).$$

$$V=\frac{8Ab}{aB}\int_0^a\sqrt{[(B^2-x^2)(a^2-x^2)]}dx,\ a<B\ldots.(2).$$

Let $x=B\sin\theta$ in (1).

Then $V=8ABb\int_0^{\frac{1}{2}\pi}\sqrt{[1-(B^2/a^2)\sin^2\theta]}\cos^2\theta d\theta$

$$=\frac{8Aa^2b}{3B}\{[1+B^2/a^2]E[B/a,\ \tfrac{1}{2}\pi]-[1-B^2/a^2]F[B/a,\ \tfrac{1}{2}\pi]\}.$$

Let $x=a\sin\theta$ in (2).

Then $V=8Aab\int_0^{\frac{1}{2}\pi}\sqrt{[1-(a^2/B^2)\sin^2\theta)}\cos^2\theta d\theta$

$$=\frac{8AB^2b}{3a}\{[1+a^2/B^2]E[a/B,\ \tfrac{1}{2}\pi]-[1-a^2/B^2]F[a/B,\ \tfrac{1}{2}\pi]\}.$$

MECHANICS.

121. **Proposed by W. J. GREENSTREET, M. A., Editor of The Mathematical Gazette, Stroud. Gloucestershire, England.**

Two equal scale pans of mass m hang at rest over a smooth pulley. An inelastic particle, mass M, is dropped from a height h into one pan, and simultaneously another of equal mass and elasticity e is dropped from the same heignt into the other. Prove that every impact occurs when the pans are in their original positions, and find the total space described by either pan before motion ceases.

Soluiion by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

The velocity of each particle just before impact$=\sqrt{(2gh)}$.

The velocity of first rebound of elastic particle$=e\sqrt{(2gh)}$.

The velocity of second rebound of elastic particle$=e^2\sqrt{(2gh)}$.

The velocity of nth rebound of elastic particle$=e^n\sqrt{(2gh)}$.

The elastic particle imparts a velocity to the scale pans at first impact $=\frac{M(1+e)}{2m+M}\sqrt{(2gh)}$, at the second impact a velocity$=\frac{Me(1+e)}{2m+M}\sqrt{(2gh)}$, at the nth impact a velocity$=\frac{Me^{n-1}(1+e)}{2m+M}\sqrt{(2gh)}$.

The inelastic particle imparts a velocity to the scale pans at first impact $=\frac{M}{2m+M}\sqrt{(2gh)}$.

Resultant velocity of these two particles at first impact$=\frac{Me}{2m+M}\sqrt{(2gh)}$.

The acceleration caused by inelastic particle$=\frac{Mg}{2m+M}$.

The time required for the scale pans to return to their original positions

$$=\frac{2Me}{2m+M}\sqrt{(2gh)}/\frac{Mg}{2m+M}=2e\sqrt{(2h/g)}.$$

The time required for the elastic particle to return to the same position $2e\sqrt{(2gh)}/g=2e\sqrt{(2h/g)}$.